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Controversies about the value of the third cosmic velocity

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ABSTRACT

The purpose of writing this article was to derivate the formula for the *third cosmic velocity* using only the laws of conservation in the heliocentric reference system. It turns out that it can be done by using elementary mathematics, thanks to which the reasoning and calculations are affordable for one interested in this subject. By the way, we wanted to discuss the errors that appear even in well-known textbooks and professional articles, whose commitment leads to incorrect results. The magnitudes of the third cosmic velocity obtained by us are: $v_{3\mathrm{average}}$ = 16.68 km/s, $v_{3\mathrm{perihelion}}$ = 16.57 km/s and $v_{3\mathrm{aphelion}}$ = 16.79 km/s.

INTRODUCTION

The ***third cosmic velocity*** is defined as the minimum initial velocity, which the object on the Earth's surface requires to escape not only from the Earth's gravitational field, but from the solar gravitational field as well. In other words, it is the initial velocity of such magnitude that the object launched from the Earth's surface at this speed can escape from the Solar System and move on to the infinity. The *third cosmic velocity* is defined with respect to the Earth.

The minimum value, of course, applies to the situation when the velocity vector of the object launched from the Earth's surface is parallel to the instantaneous orbital velocity of the Earth (Fig. 1). In the estimating the value of the *third cosmic velocity* some approximations are made: the rotation of the Earth and the interaction of the Earth's atmosphere with the launched object are neglected. The gravitational interaction between the launched object and other objects in the Solar System except the Earth and the Sun itself are neglected as well.

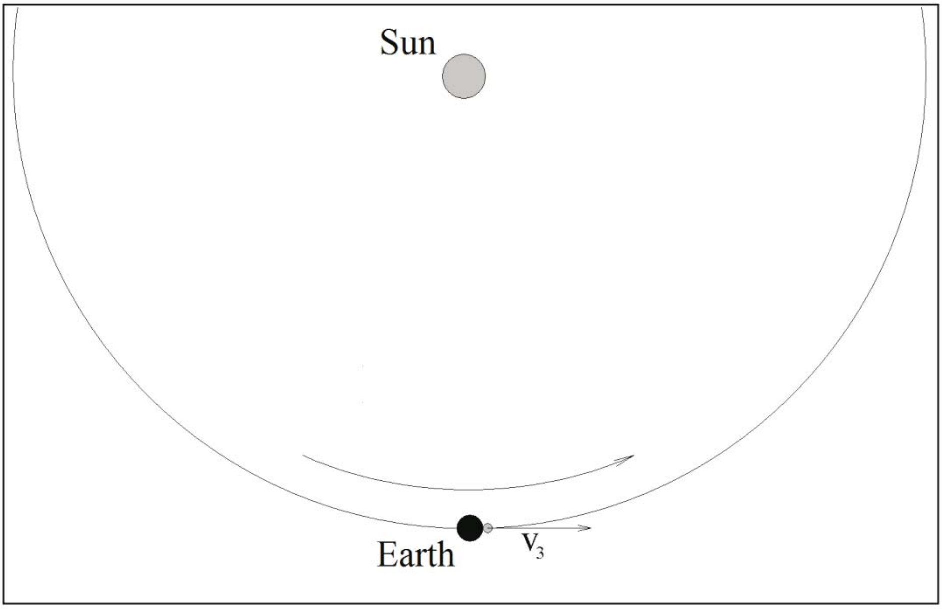

Fig. 1. Launching an object from the Earth with the *third cosmic velocity*.

Around the *third cosmic velocity* arose some controversies, misunderstandings, and even disputes concerning its value. It seems that in the twenty-first century, in the time when cosmic flights do not cause any sensation and when probes launched from the Earth reach not only the surface of our natural satellite, but also reach Mars, Jupiter and Pluto (and some of them leave the Solar System forever), the knowledge of the cosmic velocities should have been already well established and available at least for those who are interested.

While it seems all the physics textbooks for secondary schools and a number of websites for physics, astronomy or astrophysics (even an article on Wikipedia) allow the reader to thoroughly acquaint with the issue of the first and second cosmic velocity, giving a precise definition and derivation of the whole formula, whereas the majority of sources (books and websites in Polish and English) rather ignore the *third cosmic velocity* issue. Those sources which do mention on this issue usually provide a more or less clear definition (especially the Internet contains many inaccuracies), and almost always the value that is expected to be v_3 = 16.7 km/s. It is very difficult to find a complete derivation of the formula. The issue is really not trivial, because it concerns, in the most simple terms, the three bodies.

But one should be aware that by accepting some approximations, but not falsifying the physical sense and not being oversimplified, this problem can be solved without the use of numerical methods.

However it appears that the value of the *third cosmic velocity* reported by the authors of general physics academic textbooks is different. The "Physics" by Orear gives 13.5 km/s [1] whereas the "Introduction to Physics" by Wróblewski and Zakrzewski gives 16.65 km/s [2]. The difference is not due to approximations in physical constants, or approximations in some stages of calculations, but due to different way of derivation which leads to different final formula.

In 2011, an article "A Study on the Theoretical Value of the Third Cosmic Velocity" was published in the online scientific journal [3]. The authors, after conducting their own considerations (according to us wrong!) and calculations, questioned the most frequently published value of 16.7 km/s. They claim that the correct value is 13.6 km/s, which is exactly the same as in the Orear's textbook [1].

In this short paper we present a derivation of the *third cosmic velocity* formula, based only on the three laws of conservation: momentum, angular momentum, and energy in the reference frame attached to the Sun. Then we compare our reasoning with those by Orear and by Wróblewski and Zakrzewski, and by that described in the article mentioned above. We believe that for a complete understanding of our considerations the knowledge of physics and mathematics at the secondary school level is sufficient.

ANALYSIS OF THE PROBLEM

We consider the launch from the Earth's surface of an object having a mass of many orders of magnitude smaller than the mass of the Earth. This object will be called the *bullet*. Since the Earth's mass is $M = 6 \cdot 10^{24}$ kg, assuming the *bullet* mass of $m = 10^6$ kg, the mass ratio is of order $\frac{m}{M} = 10^{-18}$.

Let us introduce the term ***gravitational interaction sphere*** [2]. The *gravitational interaction sphere* will be called the area of the spherical surface centered at the center of the planet, in which the motion of any object (such as a *bullet*) is better described as a motion in the gravitational field of the planet with the perturbations from the Sun than the motion in the gravitational field of the Sun with perturbations from the planet. The radius of the Earth's *gravitational interaction sphere* is $r_0 \approx 926\,000$ km [1].

[1] We refer the careful reader to the section II.3.3 of the textbook [2], however we note that a precise determination of the r_0 radius is not necessary in our case.

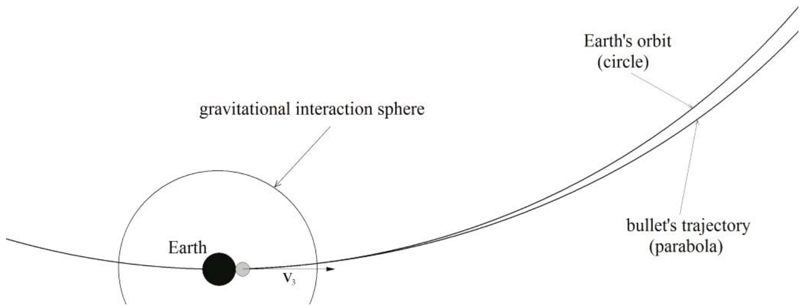

Fig. 2. The trajectory of the *bullet* after launch from the Earth with the *third cosmic velocity*.

The lowest escape velocity (with respect to the Sun) of an object moving on the Earth's orbit will be called the ***parabolic velocity***. The name comes from the fact that a trajectory such an escaping from the Earth's orbit object is a parabola, which initially converges with the orbit of the Earth, and then is more and more deflecting from it (Fig. 2). Just as the value of the second cosmic velocity is $\sqrt{2}$ times greater than the value of the first cosmic velocity, the value of the *parabolic velocity* is $\sqrt{2}$ times greater than the value of the orbital velocity of the Earth and is v_p = 42.24 km/s.

In our considerations we take into account the recoil effect of the Earth at the moment of launching of the *bullet* from the Earth's surface and so-called *gravitational pulling* effect i.e. entailing the Earth by the launched *bullet* as a result of the gravitational interaction between the *bullet* and the Earth. To simplify the discussion, we assume that the Earth's orbit is a circle of radius r = 150 000 000 km.

We will use the following notations:

M_S is the mass of the Sun,

M is the mass of the Earth,

m is the mass of the *bullet* ($m \ll M$),

r is the Earth's orbit radius,

R is the Earth's radius,

r_0 is the radius of the Earth's *gravitational interaction sphere*,

v_0 is the orbital velocity of the Earth relative to the sun v_0 = 29.87 km/s,

v_0' is the Earth's velocity relative to the sun just after the launch of a *bullet* from Earth's surface,

v_0'' is the "final" Earth's velocity relative to the sun, i.e. Earth's velocity in the moment when the receding *bullet* "stops" interact with the Earth, so when it crosses the Earth's *gravitational interaction sphere*,

v_p is the *parabolic velocity*,
v_2 is the the second cosmic velocity $v_2 = \sqrt{2G\frac{M}{R}} = 11.19$ km/s,
v_3 is the *third cosmic velocity*.

The launch of the *bullet* from the Earth's surface

Due to the fact that we consider a short interval of time (from just before till just after the launch of the *bullet* from the Earth surface), it can be assumed that at this time both the Earth and the *bullet* move along the straight line locally tangential to the orbit of the planet. In addition, if we treat the Earth and the *bullet* as the two bodies interacting only with each other, or ignore the interaction of the Sun, then the momentum of the Earth – *bullets* system has to be conserved. This leads, in the reference frame attached to the Sun, to the following equation:

$$(M+m)v_0 = Mv_0' + m(v_0' + v_3). \tag{1}$$

On the left side of the equation there is written an expression for the momentum of the system just before the launch of the *bullet* from the Earth, and on the right – just after launch it from the Earth with the *third cosmic velocity*.

Gravitational interaction launched *bullet* with the Earth and the Sun

Moving away from the Earth the *bullet* eventually crosses the Earth's *gravitational interaction sphere*. From this point we begin to treat *bullet's* motion as a motion in the gravitational field of the Sun. In order to escape from the Solar System and move to infinity, the *bullet* has to move at least with the *parabolic velocity* v_p (Fig. 3).

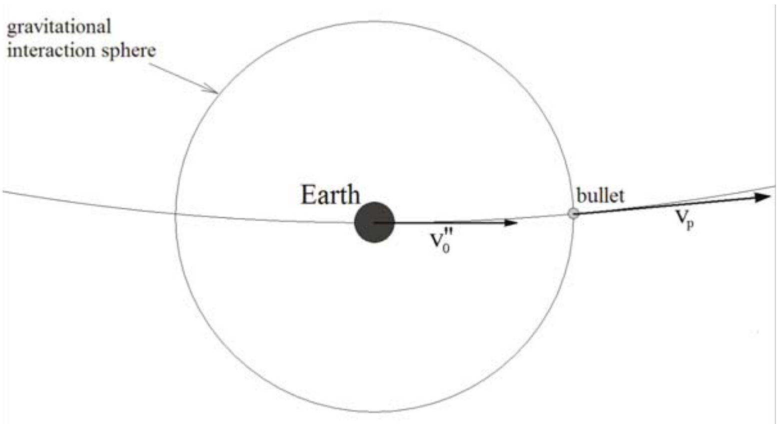

Fig. 3. The *bullet* crossing the Earth's *gravitational interaction sphere*. The *bullets* trajectory converges with Earth's orbit inside the sphere.

Here we assume that the *bullet* launched from the Earth with the *third cosmic velocity* moves along a parabola, which within the Earth's *gravitational interaction sphere* converges with the Earth's orbit (Fig. 3). This is not too coarse approximation because the diameter of the Earth's *gravitational interaction sphere* is less than 0.002 of the length of its orbit, which corresponds to an angular arc length of Earth's orbit about 0.7°. And, according to simple calculations, the distance between the points of intersection of the *gravitational interaction sphere* with the Earth's orbit and with the parabola is about 1430 km, which is less than 10^{-5} AU. Therefore, we claim that (in the reference frame attached to the Sun, according to the law of conservation of angular momentum of the Earth-*bullet* system) angular momentum at the time just after the launch of the *bullet* from the surface of the Earth is equal to the angular momentum at a time when the *bullet* leaves the Earth's *gravitational interaction sphere*:

$$Mv_0'r + m(v_0' + v_3)r = Mv_0''r + mv_pr \tag{2}$$

Similarly, for the same two moments we can write the law of conservation of energy:

$$\frac{1}{2}Mv_0'^2 + \frac{1}{2}m(v_3 + v_0')^2 - G\frac{Mm}{R} = \frac{1}{2}Mv_0''^2 + \frac{1}{2}mv_p^2 - G\frac{Mm}{r_0} \tag{3}$$

In the last equation we have neglected, on both sides, the potential energy of the Earth and the *bullet* in the Sun's gravity field, $-G\dfrac{M_S(M+m)}{r}$ as it is identical in both considered moments.

DERIVATION THE FORMULA

From equation (3) we get:

$$(v_3 + v_0')^2 = 2\mathrm{GM}\left(\frac{1}{R} - \frac{1}{r_0}\right) + \frac{M}{m}\left(v_0''^2 - v_0'^2\right) + v_p^2 \tag{4}$$

Then we see that $2\mathrm{GM}\left(\frac{1}{R} - \frac{1}{r_0}\right) = 2G\frac{M}{R}\left(1 - \frac{R}{r_0}\right) \approx 2G\frac{M}{R}$, because $\frac{R}{r_0} \approx 0{,}007$. Taking into account in equation (4) this approximation, substituting the v_2^2 for $2G\dfrac{M}{R}$ and using the difference of two squares for-mula, we have:

$$\left(v_3 + v_0'\right)^2 = v_2^{\ 2} + \frac{M}{m}\left(v_0'' - v_0'\right)\left(v_0'' + v_0'\right) + v_p^{\ 2} \tag{5}$$

Using the approximation $v_0'' + v_0' \approx 2v_0'$, we obtain[2]:

$$\left(v_3 + v_0'\right)^2 = v_2^{\ 2} + 2\frac{M}{m}\left(v_0'' - v_0'\right)v_0' + v_p^{\ 2} \tag{6}$$

Calculating from equation (2) $\frac{M}{m}\left(v_0'' - v_0'\right) = v_3 + v_0' - v_p$ and substituting to equation (6), we get:

$$\left(v_3 + v_0'\right)^2 = v_2^{\ 2} + 2\left(v_3 + v_0' - v_p\right)v_0' + v_p^{\ 2} \tag{7}$$

In equation (7) we perform multiplication and reduce similar terms. After these operations we obtain the following expression for the *third cosmic velocity*:

$$v_3^{\ 2} = v_2^{\ 2} + \left(v_p - v_0'\right)^2 \tag{8}$$

After substitution in equation (8) $v_p = \sqrt{2}v_0$ and making approximation [3] $v_0' \approx v_0$ we take the square root of both sides and obtain the final formula for the *third cosmic velocity*:

$$v_3 = \sqrt{v_2^{\ 2} + \left(\sqrt{2} - 1\right)^2 v_0^{\ 2}} \tag{9}$$

If we substitute v_2 = 11.19 km/s and v_0 = 29.87 km/s, we get the value of a *third cosmic velocity*, which turns out to be v_3 **= 16.68 km/s**.

DISCUSSION

The interaction of the Earth with the launched *bullet* may be considered in two stages. Initially, due to the recoil, the orbital Earth velocity decreases, and then increases as a result of gravitational attraction between these two bodies (*gravitational pulling*). This latter effect highly compensates for recoil so that in

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[2] Equation (1) leads to $v_0'' = \frac{m}{M}\left(v_0' + v_3 - v_p\right) + v_0'$. After substituting $\frac{m}{M} = 10^{-18}$, we get $v_0'' \approx v_0'$. However we emphasize, that there is a little difference between these velocities, which we do not neglect in equation (5).

[3] Equation (1) leads to: $v_0' = v_0 - \frac{m}{M+m}v_3 = v_0 - \frac{\frac{m}{M}}{1 + \frac{m}{M}}v_3$. Substituting $\frac{m}{M} = 10^{-18}$, we get $v_0'' \approx v_0'$.

the final result the orbital Earth speed is only insignificantly lower than the speed just before the launch of the *bullet* [4].

The value of the *third cosmic velocity* we obtained v_3 = 16.68 km/s is equal to the value reported in most of the textbook including that by Wroblewski and Zakrzewski [2], as well as the articles [4] and [5]. In the last article, the authors evaluated the *third cosmic velocity* numerically, without reference to the conservation laws, but only on the basis of general form of Newton's second law: $d\vec{p} = \vec{F}dt$.

The values v_3 = 13.6 km/s and v_3 = 13.5 km/s presented in [3] and in the textbook [1], respectively, are obtained when the *third cosmic velocity* formula is derived only on the basis of the law of conservation of energy, **assuming inexplicitly that not only the Sun, but also the Earth is stationary**. Following this approach, the energy of the *bullet* just after its launch from the surface of the stationary Earth is:

$$\frac{1}{2}mv_3^2 - G\frac{Mm}{R} - G\frac{M_S m}{r} = 0 \tag{10}$$

According to the definition of the *third cosmic velocity*, total *bullet* energy in infinity is of course zero. The value of v_3 calculated from the equation (10) is 43.6 km/s. Let us emphasize that in this reasoning the value of 43.6 km/s is relative to the Sun and to the Earth as well. However, the authors of the article [3] and Orear [1] subtract from this value the orbital velocity of the Earth, as the velocity already acquired by a *bullet* at the time of its launch, and thus get a value of respectively 13.6 km/s and 13.5 km/s. These values are lower than the value obtained by us. The explanation of this difference is simple, but not obvious. Thus, in the above reasoning, the increase of the Earth's energy (in the reference frame attached to the Sun) corresponding to an increase of Earth's speed due to the *gravitational pulling*, is completely neglected. This is unacceptable simplification because this energy is turning out to be more than twice higher than the energy of the *bullet* escaping from the Earth [4]. Only in the reference frame attached to the Earth there is no effect of *gravitational pulling*, but then the motion of the Earth with respect to the Sun cannot be ignored. Let's clearly point out here: **the energy of the *bullet* launched from the Earth has to be sufficient not only for its escape from the Solar System, but also for accelerating the Earth.**

It is worth noting that the derivation of the *third cosmic velocity* formula becomes very easy in a reference frame attached to the Earth. So that, in a natural way one can avoid the need to take into account the recoil of the Earth and

[4] The decrease of the Earth's orbital velocity due to the recoil is $v_0 - v_0' = \frac{m}{M+m}v_3$ and the following increase of the orbital velocity as a result of the *gravitational pulling* is $v_0'' - v_0' = \frac{m}{M}(v_0' + v_3 - v_p)$ Therefore, the overall change of the Earth's orbital velocity is negative: $v_0'' - v_0 \approx \frac{m}{M}(v_0' - v_p)$

the *gravitational pulling*. Such approach has been adopted by Wróblewski and Zakrzewski [2]. Let's recall that the *parabolic velocity* is the escape velocity (with respect to the Sun) from the Solar System of an object moving on the Earth's orbit far away from the Earth, so that the gravitational interaction energy with the Earth can be neglected. So the whole reasoning can be presented as follows.

In order to escape from the Solar System the *bullet* has to be launched from the Earth's surface with such a speed, which enables to cross the Earth's *gravitational interaction sphere* with at least the *parabolic velocity* (Fig. 3). This speed with respect to the Earth is $v_p - v_0 = \sqrt{2}v_0 - v_0 = \left(\sqrt{2}-1\right)v_0 =$ 12.37 km/s. If we assume that the potential energy of the *bullet* at the Earth's *gravitational interaction sphere* is zero, then the law of conservation of energy can be written as follows:

$$\frac{mv_3^2}{2} - G\frac{Mm}{R} = \frac{m\left(\sqrt{2}-1\right)^2 {v_0}^2}{2} \tag{11}$$

Since the potential energy of the *bullet* in the Sun gravity field at the moment of launching from the Earth is the same as at the moment of crossing the Earth's *gravitational interaction sphere*, in the above equation this energy is neglected, just like we did it in equation (3). After reduction of the *bullet* mass, *m*, in equation (11), multiplication by factor 2 and substitution of the square of the second cosmic velocity, ${v_2}^2$, for expression $2G\frac{M}{R}$, we get the final formula for the *third cosmic velocity*: $v_3 = \sqrt{{v_2}^2 + \left(\sqrt{2}-1\right)^2 {v_0}^2}$, which is identical as equation (9).

We see that the calculations in the reference frame attached to the Earth are simpler but require having the knowledge about the movements of objects in the heliocentric system and a mental transition from one system to the other in the derivation is necessary. In contrast, in the derivation demonstrated above we used consistently the heliocentric system only.

The *third cosmic velocity* formula can be generalized by considering the angle φ between the velocity vector of the *bullet* and the instantaneous Earth's orbital velocity [8]:

$$v_3(\varphi) = \sqrt{{v_2}^2 + \left(3 - 2\sqrt{2}\cos\varphi\right){v_0}^2} \tag{12}$$

For φ = 0°, when the *bullet* velocity is consistent with the instantaneous Earth orbital velocity, the equation (12) turns into equation (9), so v_3(0°) = 16.87 km/s. For φ = 90°, launching the *bullet* towards the Earth-Sun direction, v_3(90°) = 52.93 km/s. For φ = 180°, launching the *bullet* in reverse to the instantaneous Earth orbital velocity, v_3(180°) = 72.98 km/s.

It is worth noting that the numerical calculations performed by the authors of the article “Comparing solutions for the solar escape the problem” [5] give an interesting, non-intuitive results. According to them, the optimal direction for launching the *bullet* is not instantaneous direction of the Earth’s orbital velocity, but the direction deflected by 7° outwards of it. For such a direction numerical calculations of the *third cosmic velocity* lead to a value lower by 1.1% in comparison to that (achieved also numerically) for the direction along the Earth’s orbit. The authors do not provide a physical explanation for this unexpected result, actually they do not give any explanation or comment at all. One must take into account that their numerical calculations were based only on the general form of Newton’s second law and the assumption that the motionless Sun is at the center of the Earth’s orbit, which was artificially set as a circle. In other words, the authors as the center of the mass of the system treated the center of the Sun, they also neglected the constant movement of the Earth due to the gravitational interaction with the *bullet* (*gravitational pulling*). Such an iterative “following” along the instantaneous growth of $d\vec{p}$ vector may lead to obtaining a trajectory very close to the actual *bullet* trajectory. However not taking into account the laws of conservation (although they arise from the Newton’s laws) and using in each step of iteration approximated values (which is inherent in the numerical calculations) may lead to non-physical effects, like the deflection of escape velocity vector outwards of tangential to the Earth’s orbital velocity. There is no empirical conclusive data in that matter. The authors of mentioned paper [5] state that they calculated the velocity with accuracy closer than 0.1% .

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We get equation (9) under assumption that the Earth’s orbit is a circle, so that the Earth’s orbital velocity is constant. Thus, this formula is only an expression for the average magnitude of the *third cosmic velocity*. However, one can believe that because the Earth’s orbit is an ellipse, the *third cosmic velocity* has a different magnitude depending on the position of the Earth on the orbit, and the extreme values should occur for extreme Earth-Sun distance, which is the perihelion and aphelion. Indeed, the precise calculations lead to such conclusion. However, in order to calculate the *third cosmic velocity* in the perihelion and aphelion, it is not enough to substitute into the formula (9) respective Earth’s velocities. The elliptical shape of the orbit causes not only different Earth’s orbital velocities in the perihelion and aphelion, but also different Earth-Sun distances, and consequently different *parabolic velocities* in these extreme positions of the Earth. All of these concerns have to be taken into account already in derivation of the formula for the *third cosmic velocity*. When we take all this into consideration and that the radius of curvature of the ellipse at aphelion and/or perihelion is:

$$\rho = \frac{b^2}{a} = a\left(1 - e^2\right), \tag{13}$$

we receive the following formula for the Earth's orbital velocity at perihelion and aphelion, respectively:

$$v_{0\,peryhelion} = v_0 \sqrt{\frac{1+e}{1-e}}\,, \tag{14}$$

$$v_{0\,aphelion} = v_0 \sqrt{\frac{1-e}{1+e}} \tag{15}$$

Then the respective formulas for the *parabolic velocity* in perihelion and aphelion are as follows:

$$v_{p\,peryhelion} = v_0 \sqrt{\frac{2}{1-e}}\,, \tag{16}$$

$$v_{p\,aphelion} = v_0 \sqrt{\frac{2}{1+e}} \tag{17}$$

The symbols used in above expressions mean: a is the major semi-axis of the Earth's orbit, b is a minor semi-axis of Earth's orbit, e is eccentricity of the elliptical Earth's orbit. As a result of reasoning leading to equation (11) and using equations (14) - (17), we get the final formulas for the *third cosmic velocity* at perihelion and aphelion, respectively:

$$v_{3\,peryhelion} = \sqrt{{v_2}^2 + \frac{\left(\sqrt{2} - \sqrt{1+e}\right)^2}{1-e} \cdot {v_0}^2}\,, \tag{18}$$

$$v_{3\,aphelion} = \sqrt{{v_2}^2 + \frac{\left(\sqrt{2} - \sqrt{1-e}\right)^2}{1+e} \cdot {v_0}^2} \tag{19}$$

After substituting into them: $e = 0.0167$ [7], $v_2 = 11.19$ km/s and $v_0 = 29.87$ km/s, we get the *third cosmic velocity* magnitudes in the perihelion and aphelion, which are as follows: $\boldsymbol{v_{3perihelion}}$ **= 16.57 km/s** and $\boldsymbol{v_{3aphelion}}$ **= 16.79 km/s**.

So it turns out, perhaps a bit surprisingly, that launching the *bullet* from the Earth in order to escape from the Solar System is easier in the perihelion, where the gravitational force of the Sun is stronger than that in aphelion. Note that for $e = 0$, i. e. for the circular Earth's orbits, the formulas (18) and (19) both turn into the formula (9). We did not find the formulas (18) and (19) in the literature, but magnitudes to which they lead are confirmed by the article "Paradoxes of cosmic flights" by Ary Szternfeld [6].

In the context of equations (18) and (19) one can consider the more general problem, namely, whether the different magnitude of the *third cosmic velocity* at different positions of the Earth on its orbit is not contradictory with the law of conservation of energy. Are we not dealing with a paradox consisting in the fact that despite the constant total energy in the Earth's orbital motion, a different initial kinetic energy is necessary in order to launch a bullet beyond the Solar System ($v_{3\mathrm{peryhelion}} < v_{3\mathrm{aphelion}}$)? To explain this issue one has to be aware that in fact the Earth does not move around the Sun having a fixed position in space. We believe that accurate calculations carried out in the reference frame attached to the center of the mass of the Earth-Sun system would remove the paradox and would not lead to inconsistency with the law of conservation of energy. However, such considerations exceed the frames of this article.

Let us finally note that the Earth orbits around the Sun and it additionally rotates around its axis. The linear speed of a point on the Earth's equator due to the Earth's rotation is close to 0.5 km/s. This fact also could be used to further optimize the initial conditions in which the bullet is launched in order to leave the Solar System, just like it is made in regular flights to space.

SUMMARY

The formulas and magnitudes of the *third cosmic velocity* obtained in this paper are:

Third cosmic velocity	Formula	Magnitude [km/s]
Average	$v_3 = \sqrt{{v_2}^2 + \left(\sqrt{2}-1\right)^2 \cdot {v_0}^2}$	16.68
In peryhelion	$v_{3_{peryhelion}} = \sqrt{{v_2}^2 + \frac{\left(\sqrt{2}-\sqrt{1+e}\right)^2}{1-e} \cdot {v_0}^2}$	16.57
In aphelion	$v_{3_{aphelion}} = \sqrt{{v_2}^2 + \frac{\left(\sqrt{2}-\sqrt{1-e}\right)^2}{1+e} \cdot {v_0}^2}$	16.79

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